THE SHAPE OF THE SUNSPOT CYCLE: A ONE-PARAMETER FIT

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Abstract. Empirical description of the shape of the sunspot cycle is one of the oldest problems of solar physics. Here we show that accurate 2-pameter fit is achievable where parameters are correlated (r=0.88) for 23 solar cycles. Correlation between parameters of our fit provides the possibility of one-parameter fit if times of minima are known a' priori. One-parameter fit can also be derived from truncated dynamo models but the goodness of fit is not better than achieved for the empirical fit. We suppose that the goodness of one-parameter fit can serve as a criterion to compare different dynamo-models. One-parameter fit provides the possibility to forecast the shape of the coming cycle via forecast of one parameter which changes synchronously with the secular variation. Previous estimation of the coming decadal average sunspot number (Volobuev and Makarenko, 2008) is converted into the forecast of the shape of 24th cycle with maximum 118± 26 W. The accuracy is limited mostly with the uncertainties of the predicted secular variation and with uncertainty of the time of minimum.

Solar cycle, forecast, dynamo

1. Introduction

Minimum to minimum description of the sunspot number variation during 11-year cycle is an important problem for the long-term space weather analysis and forecast. Probably Waldmaier (1935) was the first who suggested that the variation of sunspot number during each cycle, from minimum to minimum, could be represented by one particular curve of a family of curves. He gave a graphical form of sample curves of the family. Numerous attempts were undertaken to give the mathematical description for this family. Stewart and Panofsky (1938) proposed that Pearson's type III curves

$$F = c(t-t_0)^a e^{-b(t-t_0)} \qquad (1)$$

can serve for this purpose. These curves are solution of Pearson's type III differential equation

$$\frac{\dot{F}}{F} = \frac{a}{t-t_0} - b\,. \qquad (2)$$

Here {a,b,c} are free parameters, t_0 – time of starting minimum of the cycle time $t<t_e$ where t_e – time before the ending minimum of the cycle. De Meyer (2003) customizes expression (1) for cycles (-4-22, annual data) and found that a=4; 1/b= 1.038 yr produces the smallest error of fit with free parameters c and t_0 .

Elling and Schwentek (1992) used modified F-distribution function with 5 free parameters. Hathaway, Wilson and Reichmann (1994) proposed the function

$$F=\frac{a(t-t_0)^3}{\exp\left\{\frac{(t-t_0)^2}{b^2}\right\}-c} , \qquad (3)$$

which is not a statistical distribution but produces the better results when fit is achieved using 2 – 3 free parameters only. Here, parameter $c=0.71$ was fixed for all cycles, $b = 27.12 + 25.15/(a\times 10^3)^{1/4}$ was nonlinear function of free parameter a, starting time t_0 was also considered as a free parameter. Sabarinath and Anilkumar (2008) used a binary mixture of a modified Laplace distribution

$$F = \frac{A_1}{33.2}\exp\left(\frac{-|t-t_0-41.7|}{16.6}\right)+\frac{A_2}{46}\exp\left(\frac{-|t-t_0-67.3|}{23}\right), \qquad (4)$$

with free parameters A_1 and A_2 but with a fixed table of minima t_0. This function allows them to fit the Gnevyshev's gap but causes the lack of accuracy (Figure 1). In present investigation we would address the following questions:

- Is it possible to achieve a better fit ?
- Is there a more compact function than (3) which is a solution of the differential equation like (2) but without compromising of fitting accuracy
- Is it possible to achieve the fit from truncated dynamo models?

In order to compare different fitting functions we consider the same starting time t_0 for all functions. Unavoidable uncertainty of time of minimum t_0 due to overlap of neighboring cycles causes the difficulties with exact determination of t_0 from the time series of sunspot numbers before 2-3 years of the new cycle will pass. There is still hope that it can be determined more precisely from the butterfly diagram (Wilson, 1987). On the other hand Equations (1 - 4) show that the shape of the cycle will not vary when t_0 is varied but it will have a time delay, so this time delay uncertainty is a feature of all our results. Sabarinath and Anilkumar (2008) use post-defined minima t_0 and we use here their table of minima to compare different fits.

The paper is organized as follows: Section 2 describes and discusses the data, Section 3 compares different empirical fits and argues the possibility of the one parameter fit. Section 4 considers the way to get a one-parameter fit from the truncated dynamo equations. In Section 5 we discuss the conversion of the forecast of decadal average sunspot number into the forecast of coming cycle via a one-parameter fit.

2. Data

Many problems with statistical verification of empirical rules arise from relatively short period of regular observations of solar activity. We use here the oldest index of sunspot number first proposed by Wolf and which is continued up to now as International sunspot number. Although other indices are available (Group sunspot number, Sunspot area) we do not use them here because Group sunspot number by Hoyt and Schatten (1996) is not continued up to present time whereas Greenwich Sunspot area observations start much later (1874) and provide smaller statistics of the cycles. We do not use either period 1700-1749 because monthly indices needed to compare our fit with others and because this period is most questionable as restored from rare observations. Monthly indices 1749-2009 are available at http://solarscience.msfc.nasa.gov/greenwch/spot_num.txt. The period 1749-1825 (cycles #1-6 in Zürich numeration) still has a lack of confidence particularly during anomalous cycle 4 because Schwabe starts continuous observations (>200 days per year) in 1825 only which were continued by professional observers since 1848 onward. We neglect this uncertainty for our parameterization problem because we aimed to catch the major features of the cycle which are comparable with 3-year average number whereas monthly numbers were already estimated for this period. Moreover following to Stewart and Panofsky (1938) we can speculate that cycle # 4 still belongs to the family of solar cycles because it is described with the same parameterization formulae as other cycles. So the sequence of 23 cycles with approximately 140 monthly numbers inside each are considered and the problem of fitting the shape can be solved for each cycle and for the family of the cycles.
This information is not enough to forecast the future cycle because of solving this problem requires more than 23 independent numbers only which cover 2-3 Gleissberg and one Suess cycles. This lack of experimental information can be

partly filled with using proxy information about the level of solar activity in the past. Among other proxies most confident is decadal average sunspot number derived by Solanki et al. (2004) from tree ring radiocarbon proxy and associated data. This proxy was continued up to present time by Volobuev and Makarenko (2008) and statistical prediction method was constructed to test the nonlinearity and homogeneity of the data. This proxy is considerably oversmoothed compared to the sequence of solar cycles maxima but provides additional information about secular variation of solar activity mostly about Süss and Gleissberg cycles. The data and program script for the prediction method are available at http://www.mathworks.com/matlabcentral/fileexchange/, File Id 17276.

3. Empirical Fit

Comparing the functions (1) and (2) incite us to construct the function

$R = \left(\frac{t-t_0}{T_s}\right)^{\alpha} e^{-\left(\frac{t-t_0}{T_d}\right)^2}$, which is a combination of both (1) and (2) in sense that it has a growth multiplier like (1) and decaying multiplier like (2). Following the approach by Hathaway, Wilson and Reichmann (1994) we use the Levenberg-Marquardt method to optimize free parameters. Parameter α is found close to 2, fixation α=2 leads to the function

$$R = \left(\frac{t-t_0}{T_s}\right)^{2} e^{-\left(\frac{t-t_0}{T_d}\right)^2} \qquad (5)$$

with 2 free parameters T_s and T_d (Table 1).

It is easy to check that this function is the solution of the differential equation

$$\frac{\dot{R}}{2R} = \frac{1}{t-t_0} - \frac{t-t_0}{T_d^2} \qquad (6)$$

Hathaway, Wilson and Reichmann (1994) proposed to use the χ measure to estimate the goodness of fit

$$\chi = \sqrt{\left[\sum_{i=1}^{N}(R_i - f_i)^2 / s_i^2\right] / N}, \quad (7)$$

where R_i and s_i are monthly average sunspot number and its standard deviation from daily numbers. Monthly International Sunspot Number is available with its s_i at http://solarscience.msfc.nasa.gov/greenwch/spot_num.txt . f_i – is the value of fit and N is the number of months in the cycle. Due to its construction χ measure is an error which requires a better fit for smaller cycles. We introduce a relative error

$$E_R=\chi/\chi_{<3yr>} \qquad (8)$$

to understand what is achievable goodness of the fit. Here $\chi_{<3yr>}$ is the χ - measure (7) calculated for average with 3-yr (37 months) sliding window, which we consider here as an ideal fit.

Table 1. Parameters and χ-measure for fitting function (5). χ-measures χ_2 ,χ_1– for two and one parameter fit respectively, $\chi_{<3yr>}$ - for 3-year running average. t_0 - time of minima are taken from Sabarinath and Anilkumar (2008).

Cycle No	t_0, year	T_s, years	T_d, years	χ_2	χ_1	$\chi_{<3yr>}$
1	1755.3	0.42	5.77	0.70	0.75	0.93
2	1766.5	0.23	3.92	1.36	2.31	1.31
3	1775.5	0.18	3.58	1.67	1.50	1.51
4	1784.8	0.22	4.23	0.93	0.97	0.67
5	1798.4	0.46	5.18	2.93	4.39	2.43
6	1811.0	0.57	5.75	2.66	4.07	1.70
7	1823.4	0.43	5.69	2.11	2.03	2.07
8	1833.9	0.20	3.84	0.88	0.89	0.90
9	1843.6	0.31	5.27	1.03	1.25	0.84
10	1856.0	0.30	4.80	0.92	0.87	0.78
11	1867.3	0.21	3.99	0.75	0.88	1.24
12	1879.0	0.31	4.36	2.06	3.38	2.38
13	1890.3	0.25	3.84	0.71	1.22	0.98
14	1902.2	0.34	4.56	1.16	2.02	1.10
15	1913.7	0.27	4.21	0.79	1.19	0.80
16	1923.7	0.29	4.21	0.76	1.32	0.89
17	1933.8	0.26	4.56	0.98	0.94	0.90
18	1944.2	0.21	4.27	1.20	1.04	0.99
19	1954.3	0.17	3.96	0.91	0.98	0.88
20	1964.8	0.27	4.72	0.79	0.95	0.72
21	1976.5	0.20	4.17	0.95	0.94	1.10
22	1986.8	0.17	3.73	0.82	0.95	0.98
23	1996.4	0.26	4.62	0.82	0.95	0.92

We recalculate other 2-parameter fits on the same data (Figure 1) with relative error E_R, Equation (8). It can be seen from the Figure that both the errors for fitting function by Hathaway, Wilson and Reichmann (1994) and the errors for fitting function given by Equation (5) are very close to one with average error for

23 cycles 0.97±0.19 and 1.02±0.22 respectively. The better fit for 11-year cycle is hardly possible if smaller time scale effects such as the Gnevyshev gap or quasy-biennial oscillation is not taken into account. In this respect interesting attempt to fit the Gnevyshev gap was made by Sabarinath and Anilkumar (2008) but the goodness of their fit was incorrect in their paper.

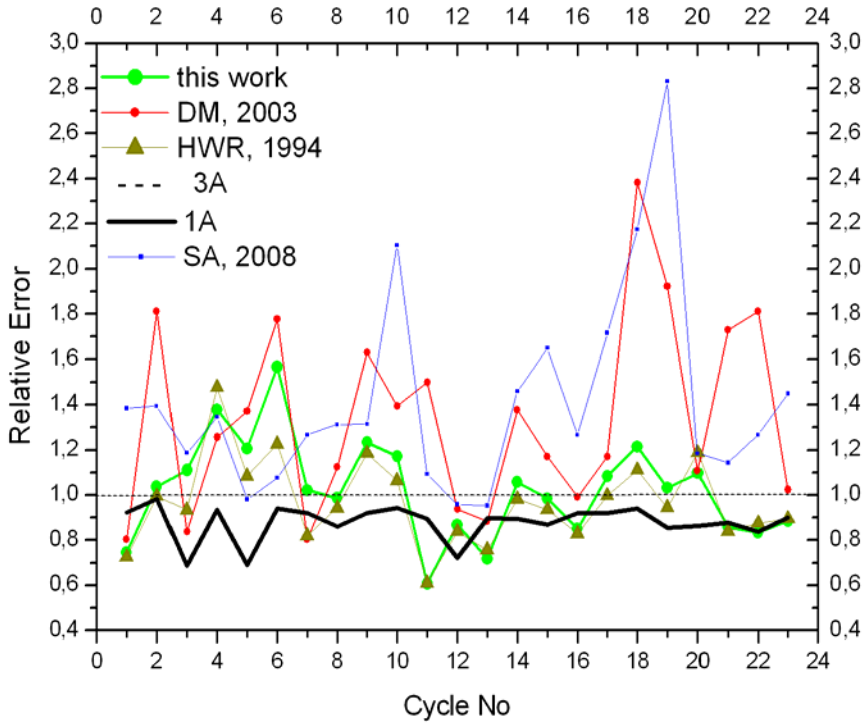

Figure 1 Comparison of 2-parameter fits. "This work" – the error of fit with fitting function given by Equation (5). "DM, 2003" - fitting function is given with Equation (1), coefficients b and c are recalculated for unified dates of sunspot minima, a=4 as adopted by De Meyer (2003). "HWR, 1994" - fitting function is given by Equation (3), coefficients a and b are recalculated for unified dates of sunspot minima, c=0.71 as were proposed by Hathaway, Wilson and Reichmann (1994). "SA, 2008" - fitting function is given by Equation (4), coefficients A_1, A_2 were adopted by Sabarinath and Anilkumar (2008). Dashed line at relative error equal one indicates the error of 3-yr sliding average used as the normalization; solid black line indicates the error of 1-yr sliding average.

Figure 1 shows that their fit is not yet perfect with average error E_R =1.41±0.44. We have found that the parameter T_d of our two-parameter fit which is given by Equation (5) significantly correlates (r=0.88, Figure 2) with parameter T_s so that the linear dependence

$$T_d = (5.7 \pm 0.7) T_s + (2.9 \pm 0.2) \qquad (9)$$

can be derived from 23 solar cycles.

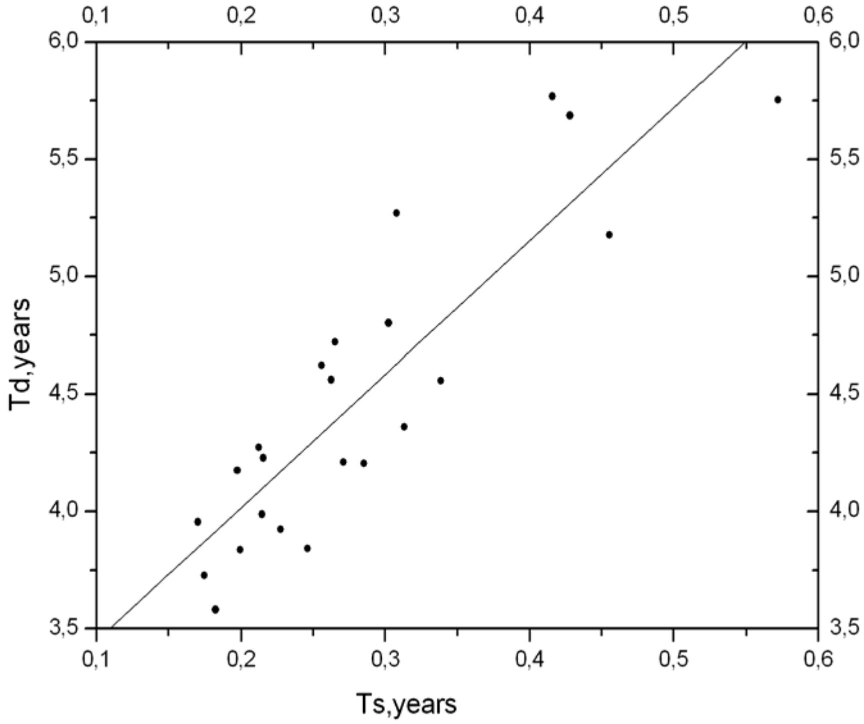

Figure 2 Dependence parameters Td on Ts for the two-parameter fit given by Equation (5).

This dependence strongly suggests the possibility of the one-parameter fit for solar cycles.

4. Parameterization from a Dynamo

It is important to find the parameterization which leads to one-parameter fit from the dynamo theory. Although consideration of total set of magnetohydrodynamic equations is difficult for this task, we can use some simplified versions of dynamo equations truncated to the set of ordinary differentional equations which are able to simulate the shape of the solar cycle. These are e.g. equations of "TOY" dynamo (Volobuev, 2006):

$$\begin{aligned} \dot{I}_p &= \tau[-\mu I_p + \omega I_t] \\ \dot{I}_t &= \tau[-\mu I_t + (\omega - A) I_p] \\ \dot{\omega} + \varepsilon\dot{\Omega} &= \tau[F_0 - I_p I_t] \end{aligned} \qquad (10)$$

In these, variables I_p, I_t may be interpreted as poloidal and toroidal fields respectively, ω – deviation of the rotation from its average profile and the constants $\{\mu=1, K=2, F_o=-11, A=\mu(K^2-K^{-2}), \tau=0.078\}$ are adopted to mimic the shape of the solar cycle, points indicate the derivatives. In the "TOY" model following to (Kitchatinov et al., 1999) is assumed that long-term variation of solar activity is mostly defined with the long-term variation in the velocity of the differential rotation *Ω(t)* of the deep layers of the convective zone which is a continuous function of time. We continue this logic here and propose that the secular variation of the sunspot number could serve as a proxy for *Ω(t)*.

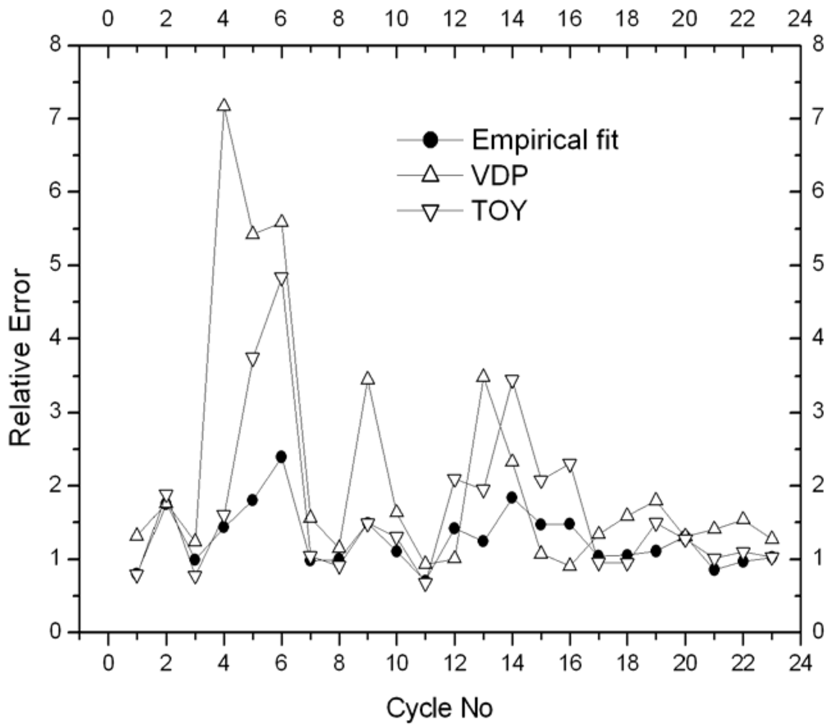

Figure 3. Relative error of the one-parameter fit. "Empirical fit" – the fitting function is given by Equations (5, 9). "VDP" – the fitting functional is derived from Mininni, Gomez and Mindlin (2001) dynamo model (12). "TOY" – the fitting functional is derived from Volobuev (2006) “TOY” dynamo model (11).

Continuous change of "slow" variable *Ω(t)* is approximated here by stepwise changes of *Ω(t)* to achieve the parameterization of the cycle. Variable *Ω(t)* has a jump at the minimum of cycle and is a constant till the minimum of the next cycle. The differential $\dot{\Omega}(t)$ of the stepwise function is a sequence of δ - functions at the minima of the cycles. It is equivalent to varying of initial conditions for $\omega(t_0)$ at the minimum of each cycle. Equation (10) takes the shape:

$$\begin{aligned} \dot{I}_p &= \tau[-\mu I_p + \omega I_t] \\ \dot{I}_t &= \tau[-\mu I_t + (\omega - A) I_p] \qquad (11) \\ \dot{\omega} &= \tau[F_0 - I_p I_t] \end{aligned}$$

We use Equation (11) to parameterize the cycle with initial conditions fixed $I_p(t_0)$ = 3, $I_t(t_0)$ =-1 for the poloidal and toroidal field respectively at the point of minimum t_0. Initial conditions for the rotation $\omega(t_0)$ was found to fit the shape of each cycle, this is a free parameter. Relative error of this one-parameter fit seems to be higher compared with the error of empirical fit (Figure 3).

Minnini, Gomez and Mindlin. (2001) show that a classical set of dynamo equations under several assumptions can be truncated to the equation of Van-der-Pol oscillator.

$$\begin{aligned} \dot{B} &= u, \\ \dot{u} &= -\varpi^2 B - \mu(3\xi B^2 - 1), \end{aligned} \qquad (12)$$

Here $\varpi = 0.2993, \mu = 0.2044, \xi = 0.0102$ are constants defined from observable time series *B(t)* which is the toroidal field and $B(t)^2$ is the sunspot number.

Varying of the initial condition $u(t_0)$ and fixing $B(t_0)=1.18$ in the minimum leads to the successive one-parameter fit for the family of cycles similar to that of "TOY" model (Figure 3).

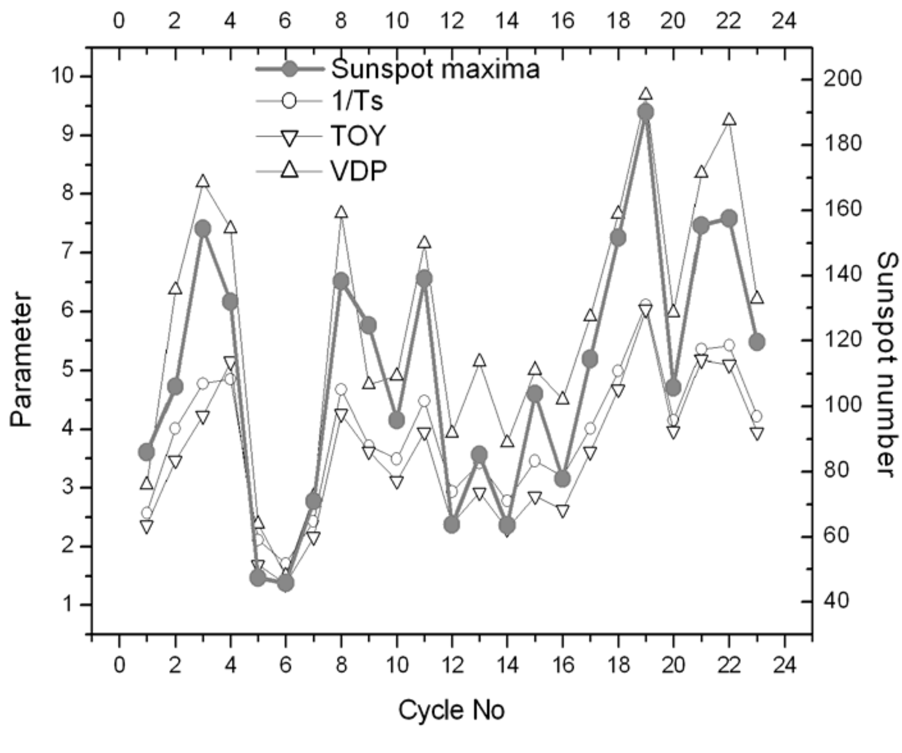

Figure 4. Cycle-to-cycle variation of the free parameter for one-parameter fit. "Sunspot maxima" – annual maximum for each cycle. "1/Ts" – free parameter for empirical fit, Equations (5, 9). "TOY" – free parameter $\omega(t_0)$ for "TOY" model fit, Equation (11). "VDP" – free parameter $u(t_0)$ for Van-der-Pol oscillator model, Equation (12).

Free parameters of empirical fit and parameters of theoretical fits vary synchronously with varying of sunspot maxima (Figure 4). Despite the fact that theoretical fits have more coefficients and are respectively more sophisticated functions than empirical fit, they do not provide a better accuracy (Figure 3). This means that the error of one-parameter fit can serve as an efficient tool to compare different dynamo models. Considering any model like a "black box" with input and output we can still verify it if we reduce the number of free parameters to 1 and calculate the error of fit. It is also an interesting task for the theory to truncate the complete system of dynamo equations and to get Equation (6).

5. From One-parameter Fit to Forecast

We consider here the Gleissberg cycle as an independent phenomenon, not a byproduct of the chaos in the Schwabe cycle. This is an assumption because "slow" changes (60 - 130 years) in the Gleissberg cycle are not separated perfectly from the "quick" (8 - 14 years) changes in the Schwabe cycle. This assumption was partially confirmed by the "TOY" model (Volobuev, 2006) which is able to mimic the shape of both the long-term variation and Schwabe cycle. It is also confirmed by the possibility of one-parameter fit and by the fact that variation of the free parameter repeats the maximum-to-maximum variation of successive cycles. Despite of the fact that this hypothesis is not a theory we can ground a forecast method on it. The probability of the forecast will help to quantify a part in variability of solar maxima which is predefined by secular variations.

Proposed forecast consists of several steps. The first step is to define the time of minimum. Then we need to forecast the decadal average of the long-term variation and rescale this quantity to the free parameter of the one-parameter fit. Evaluation of one-parameter fit gives the shape of the future cycle.

This construction requires knowing the time of minimum before it passes. Strictly speaking, nothing is known about exact position of the minimum because of well known intersection between ending cycle and coming cycle which lead to the simultaneous appearance of sunspots belonging to different cycles. While adopting the forecast technique we set to zero the unavoidable error which delays the future cycle although with the same shape. This is achieved by fixing some standard table of minima e.g. the table by Sabarinath and Anilkumar (2008). We use here the estimation by Wilson (1987) for the position of the minimum to forecast the shape of the cycle #24, namely the fact that the minimum occurs 1.4 year after the first spots of the new cycle have appeared. First spots of cycle #24 appeared on January 4, 2008, consequently, the minimum should occur approximately on May, 2009 ± 1.1 yr. We renewed here approach by Wilson (1987) via adding recent data and considering dependence of delay time on the intensity of the cycle (Figure 5). Delay time tends to decrease if previous cycle was higher (Rmax>90) or smaller (Rmax<90) than average whereas middle cycles have a maximum delay time. Assuming this rule as a hypothesis we can confirm the estimation for the current minimum on May, 2009 ± 0.7 yr. Maximum for the cycle 22 $R^{1/2}$ = 12.2 corresponds to delay Tl=0.9± 0.7 yr and to minimum on February 1997

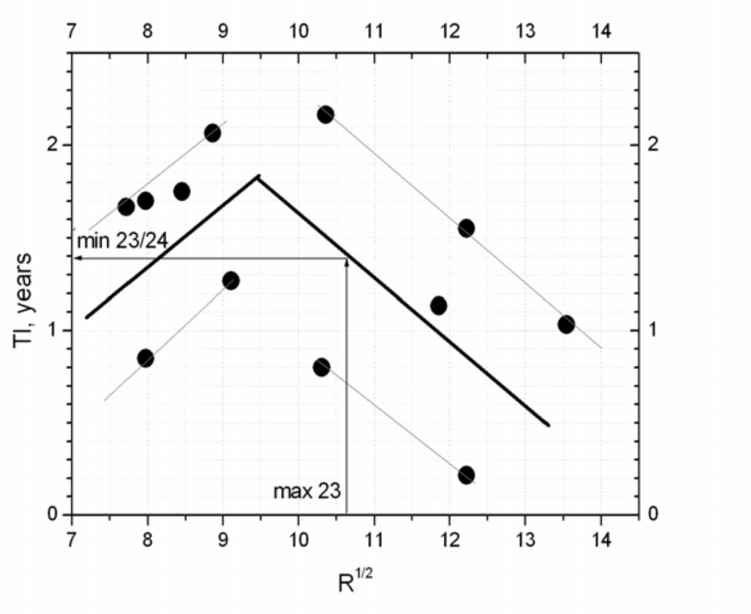

Figure 5. Delay of the minimum Tl=Em-Ef as a function of the square root of the preceding maximum sunspot number $R^{1/2}$ for cycles #12-23. Em and Ef are times of the minimum and of the appearance of the first spot of new cycle respectively.

The decadal average sunspot number dataset has one point per decade due to specificity of the radiocarbon proxy. So we will have 2 estimations of the decadal number at the minimum of each cycle: number C_0 is forecasted from last complete decadal number and C_i is estimated from current incomplete decadal number. Incomplete number C_i is estimated with strictly same spline smoothing algorithm which was used to construct the recent part of the composite time series from annual Group sunspot number and International sunspot number (see Volobuev and Makarenko, 2008). Typical is the situation for the cycle #24: we can estimate incomplete number centered at 2005 after 8 observed annual numbers and the number forecasted from the 1995. We propose that a weighted sum $C_w = 0.2 \times C_0 + 0.8 \times C_i$ would be suitable for the estimation of actual number. Generalization for any cycle is $C_w = k_1 \times C_0 + k_2 \times C_i$, where k_1=*N/10*, *N* is the number of passed years of the incomplete decade, k_2=1-k_1; This number C_w is considered as the last decadal number to forecast the next decadal number C_f with the algorithm (Volobuev and Makarenko, 2008). Then we evaluate the cycle-to-cycle forecast and estimate the free parameter T_s in the empirical fit (Equations (5, 9)) from its dependence on the forecasted number C_f for each cycle.

$$1/T_s = (2.45 \pm 0.32) + (0.047 \pm 0.009) \times C_f \qquad (13)$$

Equations (5, 9, 13) are completed for all 23 cycles. They allow successive estimation of the future solar cycle shape, e.g. its maximum (Figure 5). The correlation between the actual and predicted sunspot number maxima is not high but significant (*r=0.72*) with standard deviation σ=25.6 for all 23 cycles. The actual maximum is considered here as the maximum of monthly International sunspot number smoothed with 3 yr (37 months) running window.

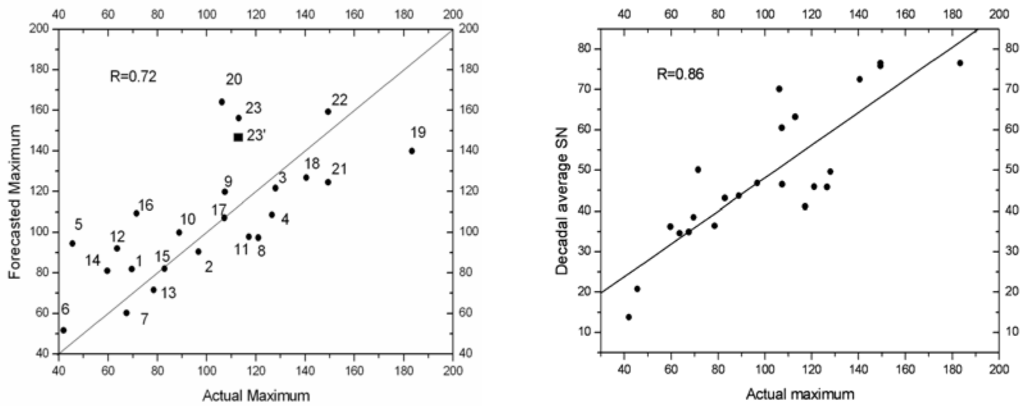

Figure 6. Successive forecast of the future maximum which starts from the known current minimum for 23 cycles (Left panel) and the major source of the forecast error (right panel). "Actual maximum" is the maximum of 37 month average international sunspot number. "Decadal average SN" – smoothed sunspot number with smoothness fitted to tree ring radiocarbon proxy-based decadal sunspot numbers (Volobuev and Makarenko,2008)

The major source of the error is some over-smoothing in the decadal radiocarbon proxy of the sunspot number and errors of the proxy itself (see Volobuev and Makarenko, 2008 for the details). Other sources of the error are: prediction error for the decadal average sunspot number and parameterization error. Both of them are considerably smaller with correlations 0.97 and 0.99 respectively. Predicted parameter T_s for cycle #24 is 0.235 which is most close to the cycle 2. The cycle

#24 sunspot maximum is predicted to be 118±26 (Figure 6), which is in close agreement to Bhatt, Jain and Aggarwal (2009). The shape of the past cycle #23 (Figure 6) is predicted from its forecasted minimum as well as the shape of the future cycle #24.

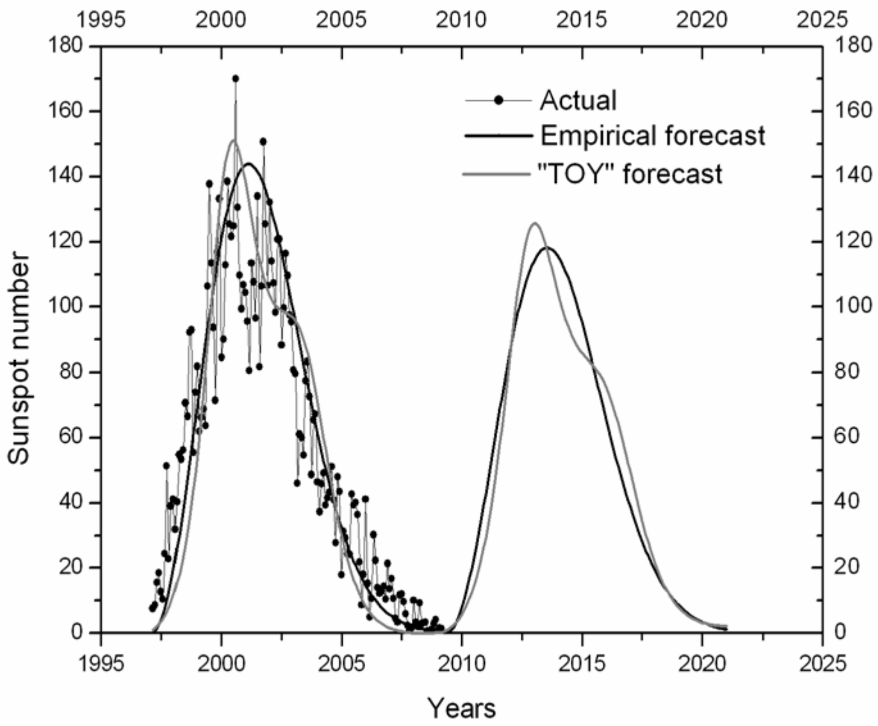

Figure 7. Predicted shape of the cycles #23-24. "Actual" – International monthly sunspot number. "Empirical forecast" – uses empirical one-parameter fit, Equations (5, 9). "TOY" forecast – uses one-parameter fit with the functional derived from the "TOY" dynamo Equation (11).

Proposed forecast is not most accurate if compared with other methods e.g. with Ohl "precursor" -based methods, but it is probably most confident because it is based on the statistics of decadal average sunspot number during the Holocene (Volobuev and Makarenko, 2008). Considering the huge number of predictions available for the maximum of cycle #24 (see e.g. reviews by Kane (2007) and Pesnell (2008)) we can conclude that this problem is not yet solved now. Specifically Dikpati, de Toma and Gilman (2006), predict the new cycle as extremely high whereas Svalgaard, Cliver, and Kamide (2005) predict it as extremely small. In this context our approach can serve to reduce scattering of possible predictions. On the other hand proposed method is independent of any "precursor" method and the combination of the methods may lead to better result than each of them.

Conclusions

A new function is proposed to fit the family of solar cycles. This function has similar error of fit as the function by Hathaway, Wilson and Reichmann (1994), but it has linear dependence between its two free parameters, consequently, it is more suitable for one-parameter fit. Based on this function, a new method of 11-year cycle forecast from its time of minimum is proposed on the base of the tree ring radiocarbon proxy for solar activity in the past. Base hypothesis of the method is that long-term changes (envelope) of solar activity predefine the maximum of the future cycle. We conclude that:

- At least 70% of the maximum of the cycle is predefined by a long-term trend.
- The error of one-parameter fit of the solar cycle shape can be used as a test for comparison of dynamo models.
- The cycle #24 maximum is predicted 118±26.

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References

Bhatt, N.J., Jain, R., Aggarwal, M.: 2009, *J. Astrophys. Astr.* **30**, 71.

De Meyer, F..: 2003, *Solar Phys.*, **217**, 349.

Dikpati, M., de Toma, G., Gilman, P.A.: 2006, *Geophys. Res. Lett.* **33**, L05102.

Elling, W., Schwentek, H.:1992, *Solar Phys.* **137**, 155.

Hathaway, D.H., Wilson, R.M., Reichmann, E.J.: 1994, *Solar Phys.* **151**, 177.

Hoyt, D.V., Schatten, K.H.: 1996, *Solar Phys.* **165**, 181.

Kane, R.P.: 2007, *Solar Phys*. **243**, 205.

Kitchatinov, L.L., Pipin, V.V., Makarov, V.I., Tlatov, A.G.: 1999, *Solar Phys.* **189**, 227.

Mininni, P.D., Gomez, D.O., Mindlin, G.B.: 2001, *Solar Phys*., **201**, 203.

Pesnell, W.D.:2008, *Solar Phys*., **252**, 209

Sabarinath, A., Anilkumar, A.K.: 2008, *Solar Phys.* **250**, 183.

Solanki, S.K., Usoskin, I.G., Kromer, B., Schüssler, M., Beer, J.: 2004, *Nature* **431**, 1084.

Stewart, J.Q., Panofsky, A.A.:1938, *Astrophys. J.*, **4**, 385.

Svalgaard, L., Cliver, E.W., Kamide, Y.: 2005, *Geophys. Res.Lett.* 32, L01104.

Volobuev, D.M., Makarenko, N.G.: 2008, *Solar Phys.*, **249**, 121.

Volobuev, D.: 2006, *Solar Phys.*, **238**, 421.

Waldmeier , M. : 1935, *Astr. Mitt*., **133**, 105.

Wilson, R.M.: 1987, *Solar Phys.*, **111**, 255.